A Hybrid Quantum-Classical Algorithm for Robust Fitting

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Robust fitting via consensus maximisation

Outliers exist in computer vision

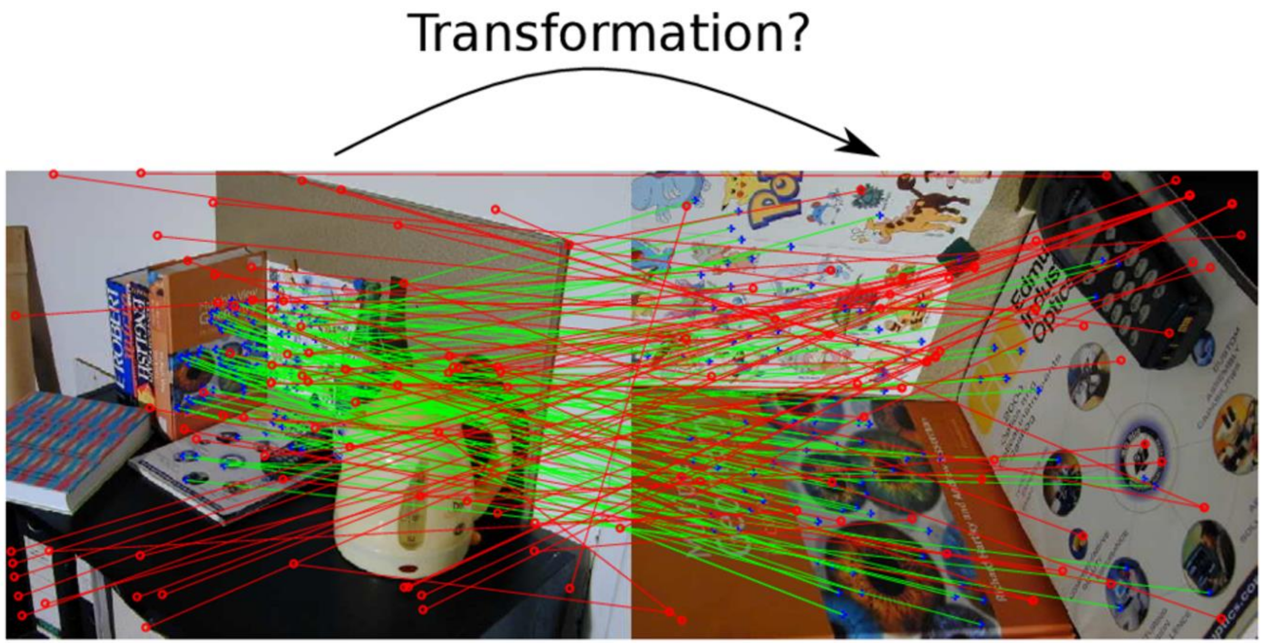


Figure 1 : Feature point matches containing outliers (red lines).

Least square is sensitive to outliers

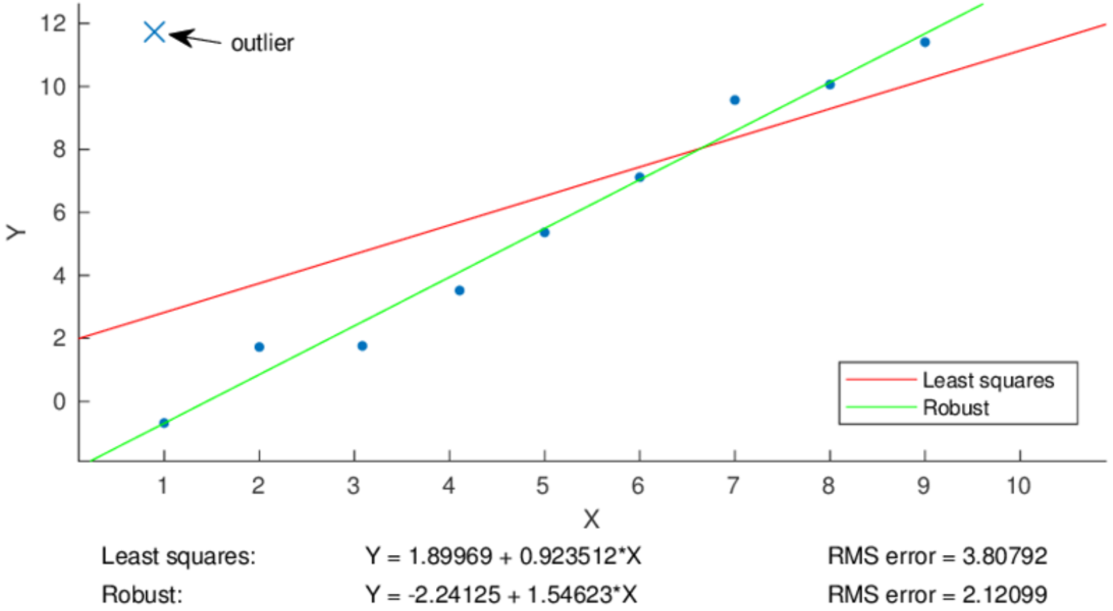
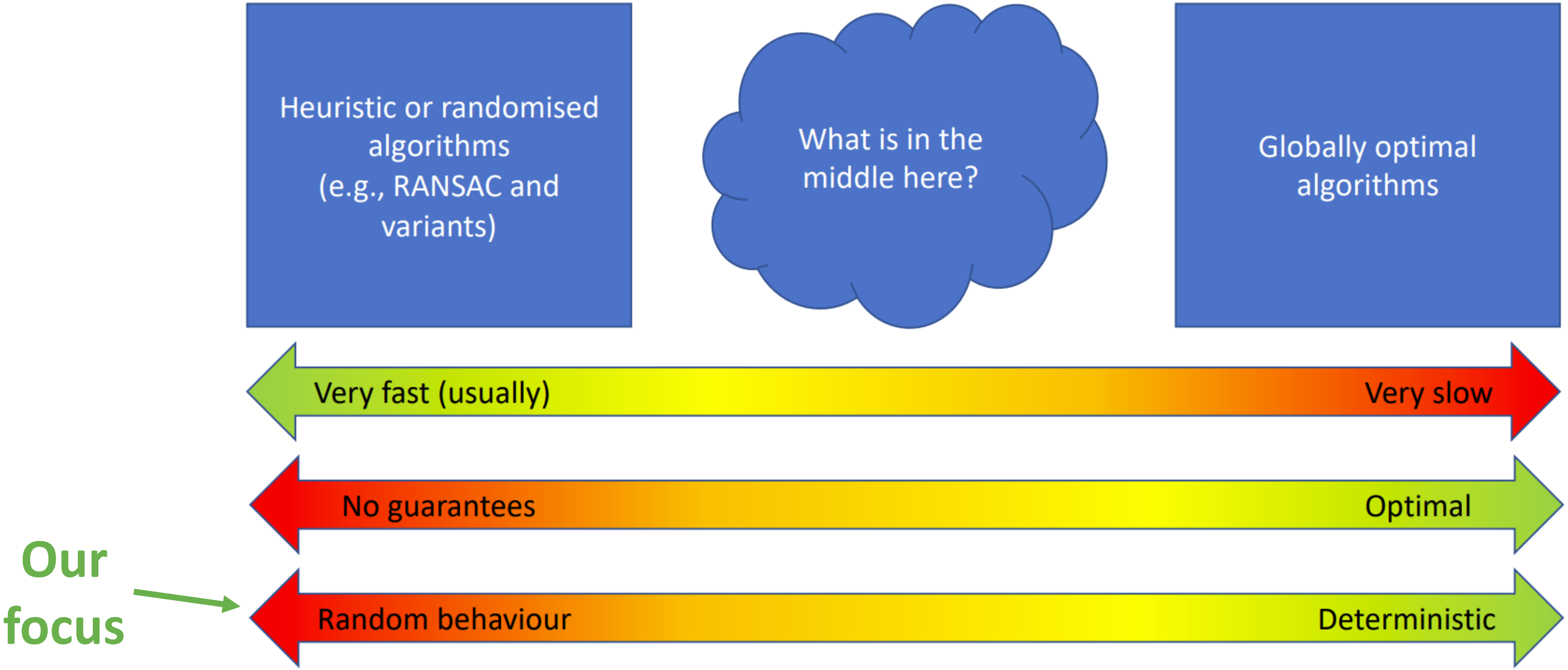


Figure 2 : Sensitivity of least squares to outliers.

Objective: make model fitting robust (insensitive) to outliers

Research gap in consensus maximisation

Theoretical results on classical computers [1]

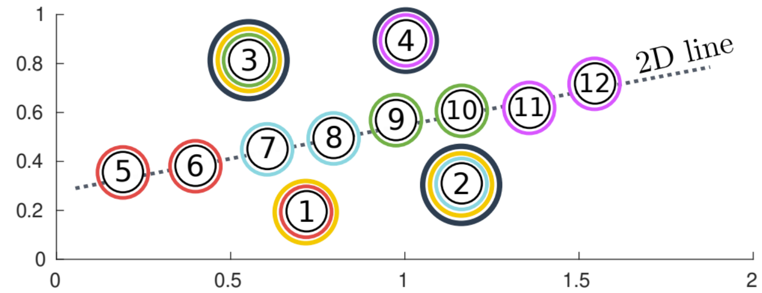


Motivation		**Contributions**
1. Investigate a new type of computer	→	A *hybrid quantum-classical algorithm*
2. Address the random behaviour	→	An *error bound* $\lvert\mathcal{I}^*\rvert - \lvert\tilde{\mathcal{I}}\rvert \leq \rho$

[1] Tat-Jun Chin, Zhipeng Cai, and Frank Neumann. “Robust fitting in computer vision: Easy or hard?.” ECCV 2018.

Maximise consensus as minimise vertex cover

2D data space



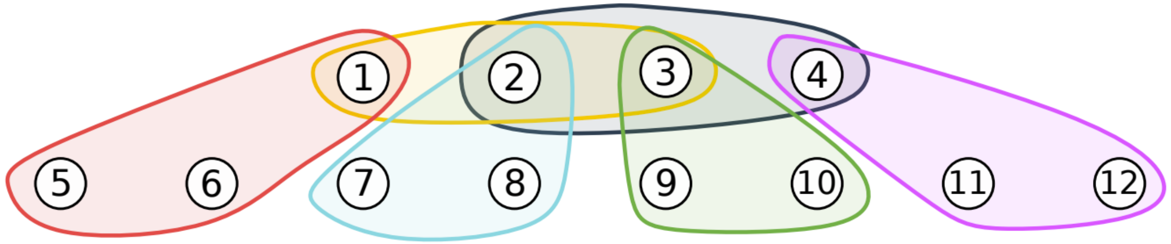
Infeasible bases

- $\{1, 5, 6\}$
- $\{2, 7, 8\}$
- $\{3, 9, 10\}$
- $\{4, 11, 12\}$
- $\{1, 2, 3\}$
- $\{2, 3, 4\}$

Outlier set $\{1, 2, 3, 4\}$

Consensus set $\{5, 6, 7, 8, 9, 10, 11, 12\}$

Hypergraph space



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Hyperedges

Vertex cover $\{1, 2, 3, 4\}$

Independent set $\{5, 6, 7, 8, 9, 10, 11, 12\}$

Maximise consensus as minimise vertex cover

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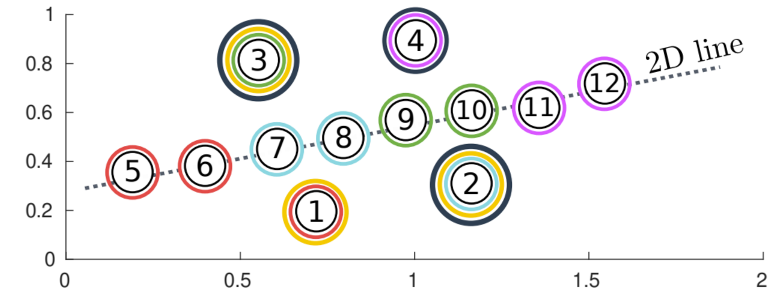
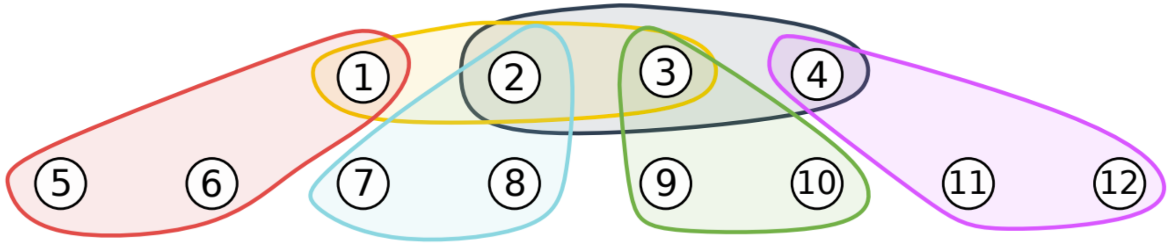


Hypergraph space



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Maximum consensus set ⟺ **Maximum independent set**

⇕

Minimum outlier set ⟺ **Minimum vertex cover**

Hybrid quantum-classical robust fitting

Hypergraph vertex cover as 0-1 ILP

$$I(A) = \min_{\mathbf{z} \in \{0,1\}^N} \|\mathbf{z}\|_1 \quad s.t \quad \mathbf{A}^T\mathbf{z} \geq 1_M,$$

where $z_i = 1$ implies vertex $i \in$ vertex cover

can be solved by quantum annealer

Hypergraph vertex cover as QUBO

$$Q_\lambda(A) = \min_{\mathbf{v} \in \{0,1\}^{N+\delta' M}} [\mathbf{v}^T \quad 1](\mathbf{J} + \lambda \mathbf{H}_A^T \mathbf{H}_A)[\mathbf{v}^T \quad 1]^T$$

where, $\mathbf{v} = \left[\mathbf{z}^T \quad \mathbf{t}_{(1)}^T \quad \ldots \quad \mathbf{t}_{(M)}^T\right]$

Hybrid quantum-classical robust fitting algorithm

1. $A \longleftarrow$ Sample new hyperedge
2. Decay penalty λ
3. Solve $Q_\lambda(A)$ using *quantum annealing*
4. If $\mathcal{I} \leftarrow \mathcal{V} \backslash C_{\mathbf{z}}$ is a consensus set
5. If $\|\mathbf{z}\|_1 < \|\mathbf{z}_{\text{best}}\|_1$, then
6. $\mathbf{z}_{\text{best}} \leftarrow \mathbf{z}$ and $\mathcal{I}_{\text{best}} \leftarrow \mathcal{I}$
7. Repeat step 1

Hybrid quantum-classical robust fitting

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Relaxation

$$LP(A) = \min_{\mathbf{z}\in[0,1]^N} \|\mathbf{z}\|_1 \quad s.t \quad \mathbf{A}^T\mathbf{z} \geq 1_M,$$

Hybrid quantum-classical robust fitting algorithm

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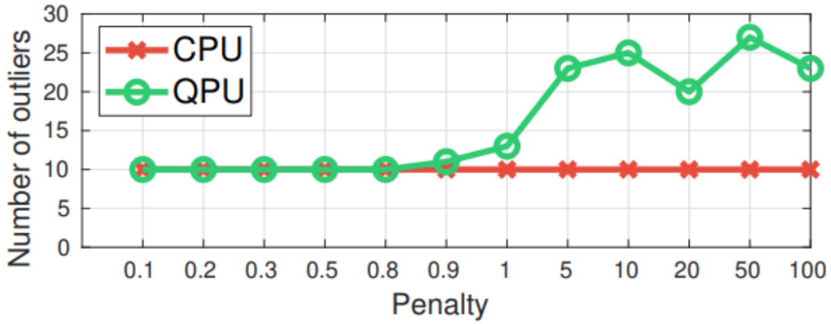
Error bound

$$|\mathcal{I}^*| - |\mathcal{I}_{\text{best}}| \leq \|\mathbf{z}_{\text{best}}\|_1 - LP(A)$$

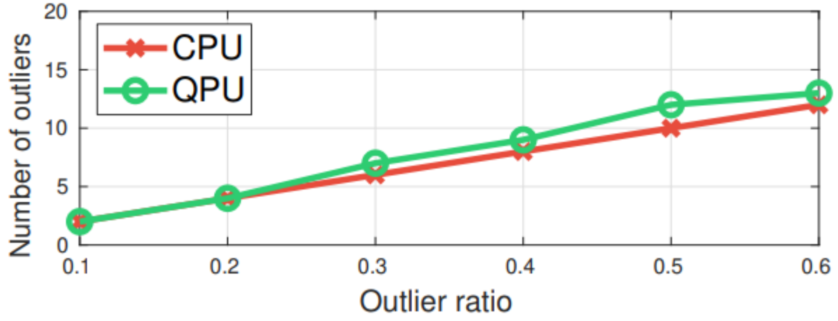
Current best solution

Relaxation

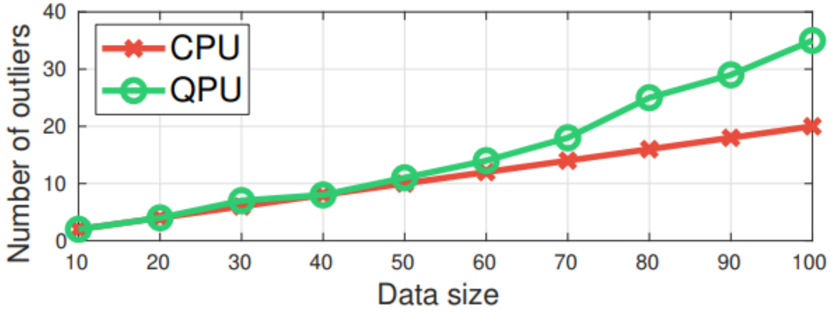
Robust linear regression on synthetic data



(a) Effect of penalty λ ($N = 50$, outlier ratio $= 0.2$)



(b) Effect of outlier ratio ($N = 20$, $\lambda = 1.0$)



(c) Effect of data size N ($\lambda = 1.0$, outlier ratio $= 0.2$)

Figure 1. Comparison between CPU and QPU in solving QUBO

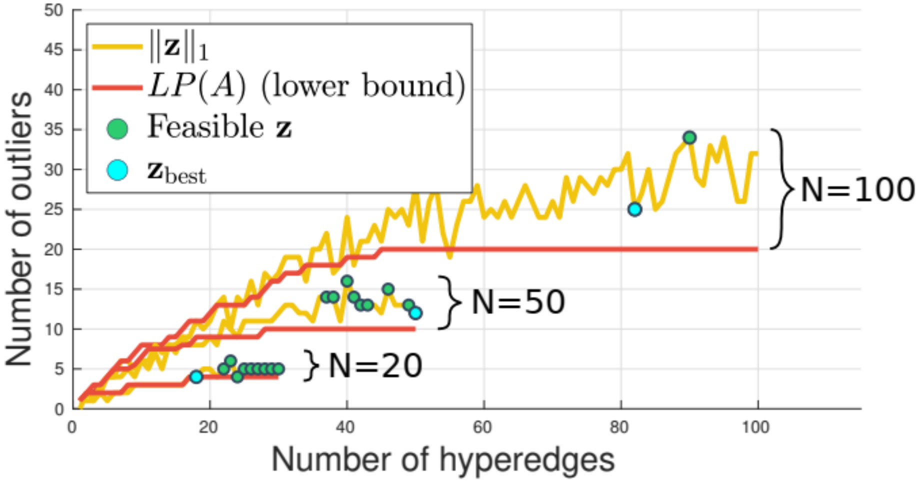


Figure 2. Number of outliers $\|\mathbf{z}\|_1$ optimised by QPU and lower bound $LP(A)$, plotted across the iterations of the proposed algorithm

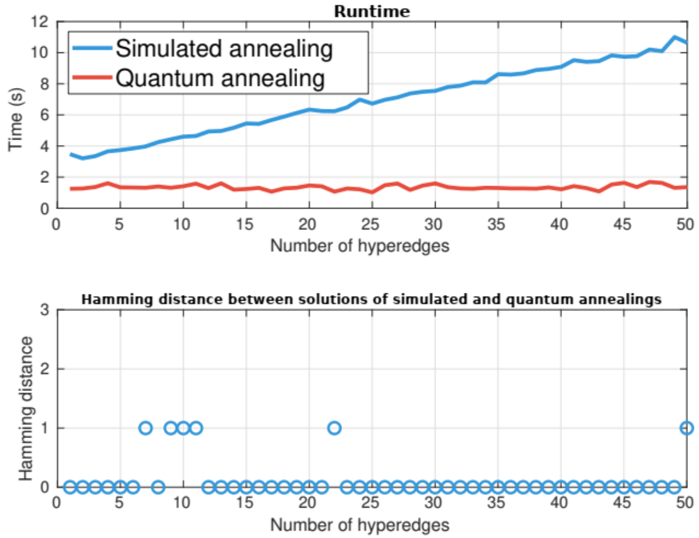


Figure 3. Comparison between quantum annealing (on D-Wave Advantage) and simulated annealing (on classical computer)

Fundamental matrix estimation

- Solve QUBO with simulated annealing
- Alg. 1-F we run the algorithm with 300 iterations
- Alg. 1-E we terminate the algorithm as soon as a consensus set is found

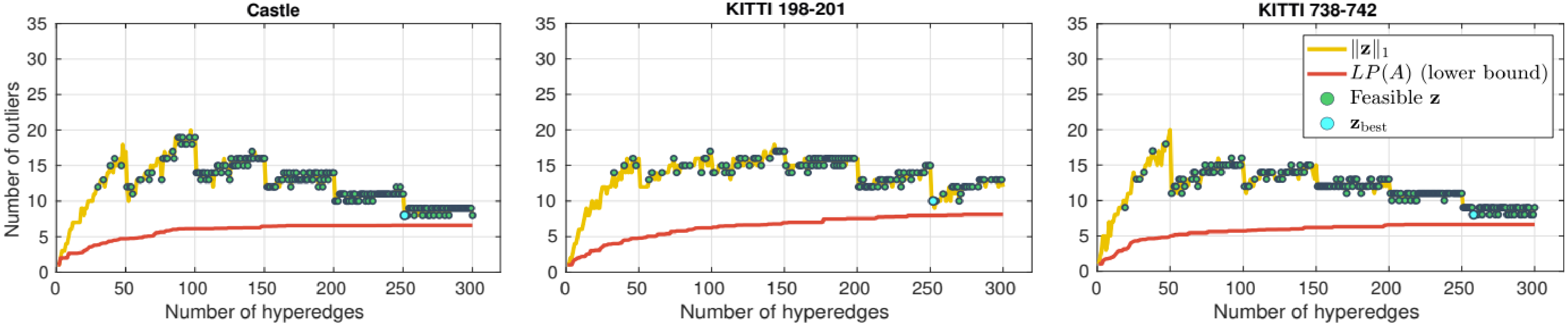


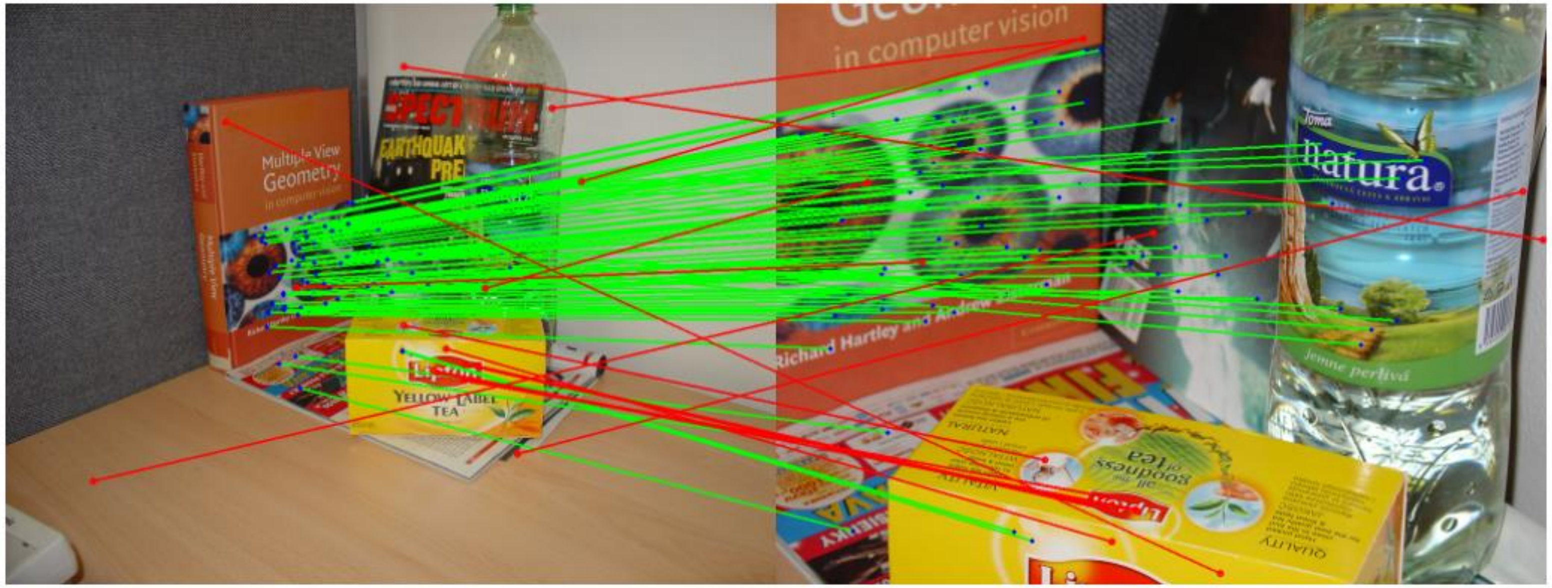
Figure 4. Fundamental matrix estimation, where number of outliers $\|\mathbf{z}\|_1$ and lower bound $LP(A)$ plotted across the iterations of Alg. 1-F

Method		RS [36]	LRS [21]	FLRS [47]	EP [45]	IBCO [15]	QRF [20]	Alg. 1-E	Alg. 1-F
Castle	$\lvert\mathcal{I}\rvert$ (Error bound)	74 (–)	74 (–)	74 (–)	70 (–)	76 (–)	73 (–)	72 (8.17)	76 (1.41)
$N = 84$	Time (s)	0.20	0.11	0.20	0.25	0.34	199.48	18.07	1998.87
Valbonne	$\lvert\mathcal{I}\rvert$ (Error bound)	34 (–)	36 (–)	36 (–)	33 (–)	38 (–)	29 (–)	36 (6.00)	36 (4.00)
$N = 45$	Time (s)	0.21	0.20	0.31	0.34	0.44	110.30	6.71	1915.82
Zoom	$\lvert\mathcal{I}\rvert$ (Error bound)	90 (–)	91 (–)	91 (–)	92 (–)	95 (–)	89 (–)	93 (9.91)	94 (3.64)
$N = 108$	Time (s)	0.31	0.29	0.14	0.21	0.35	257.03	92.35	2109.13
KITTI 104-108	$\lvert\mathcal{I}\rvert$ (Error bound)	309 (–)	313 (–)	312 (–)	318 (–)	321 (–)	256 (–)	320 (9.91)	324 (2.30)
$N = 337$	Time (s)	0.04	0.04	0.07	0.28	0.39	799.33	137.26	2408.04
KITTI 198-201	$\lvert\mathcal{I}\rvert$ (Error bound)	306 (–)	308 (–)	307 (–)	308 (–)	312 (–)	309 (–)	308 (10.00)	312 (1.89)
$N = 322$	Time (s)	0.05	0.13	0.07	0.23	0.42	774.06	36.15	2350.39
KITTI 738-742	$\lvert\mathcal{I}\rvert$ (Error bound)	481 (–)	483 (–)	483 (–)	491 (–)	492 (–)	447 (–)	492 (5.88)	493 (1.39)
$N = 501$	Time (s)	0.05	0.18	0.23	0.53	0.61	1160.12	22.46	2506.04

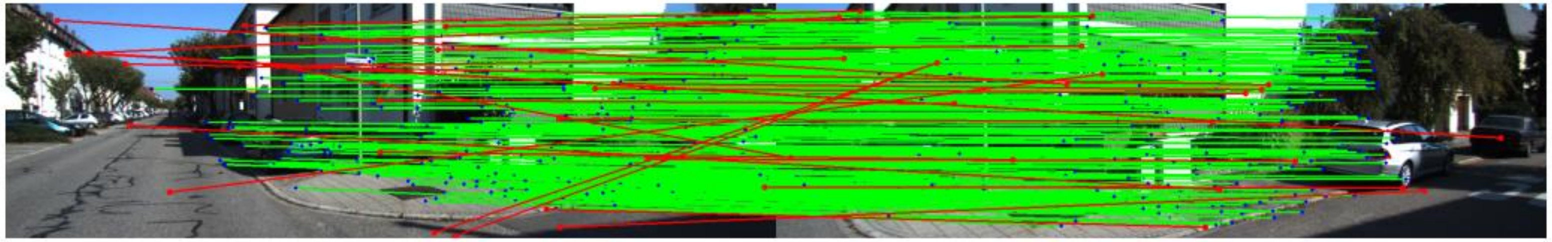
Alg. 1-F provides tighter error bound & higher quality solution

Table 1. Fundamental matrix estimation. Only our algorithm amongst all methods returns error bounds

Fundamental matrix estimation



(a) Zoom



(b) KITTI 104-108

Figure 5. Qualitative results on fundamental matrix estimation

Multi-view triangulation

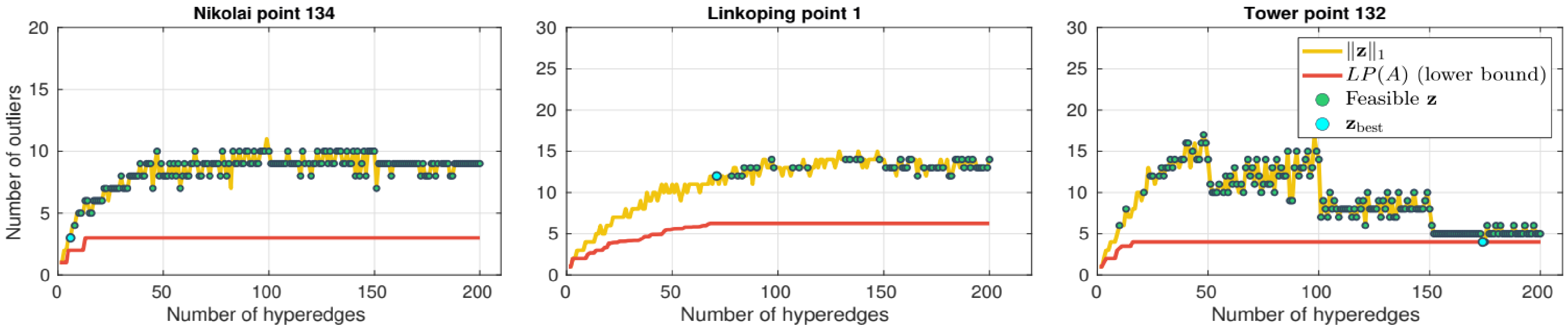


Figure 6. Multi-view triangulation, where number of outliers $\|\mathbf{z}\|_1$ and lower bound $LP(A)$ plotted across the iterations of Alg. 1-F

Method		RS [36]	LRS [21]	FLRS [47]	EP [45]	IBCO [15]	QRF [20]	Alg. 1-E	Alg. 1-F
Nikolai point 134	$\lvert\mathcal{I}\rvert$ (Error bound)	21 (–)	21 (–)	21 (–)	21 (–)	21 (–)	21 (–)	21 (1.00)	21 (0.00)
$N = 24$	Time (s)	0.24	0.32	0.30	0.34	0.36	158.39	6.12	159.28
Nikolai point 534	$\lvert\mathcal{I}\rvert$ (Error bound)	16 (–)	16 (–)	16 (–)	15 (–)	17 (–)	17 (–)	16 (2.00)	16 (1.00)
$N = 20$	Time (s)	0.27	0.35	0.25	0.29	0.32	154.63	8.71	147.14
Linkoping point 1	$\lvert\mathcal{I}\rvert$ (Error bound)	15 (–)	15 (–)	15 (–)	14 (–)	16 (–)	14 (–)	13 (5.75)	13 (5.75)
$N = 25$	Time (s)	0.25	0.30	0.34	0.38	0.47	175.83	20.05	153.79
Linkoping point 14	$\lvert\mathcal{I}\rvert$ (Error bound)	36 (–)	36 (–)	36 (–)	35 (–)	37 (–)	32 (–)	37 (4.67)	37 (4.27)
$N = 52$	Time (s)	0.27	0.44	0.38	0.53	0.64	360.37	130.10	194.46
Tower point 3	$\lvert\mathcal{I}\rvert$ (Error bound)	73 (–)	73 (–)	73 (–)	73 (–)	73 (–)	73 (–)	72 (3.00)	72 (1.00)
$N = 79$	Time (s)	0.28	0.64	0.32	0.36	0.43	555.27	27.26	177.43
Tower point 132	$\lvert\mathcal{I}\rvert$ (Error bound)	79 (–)	79 (–)	79 (–)	79 (–)	81 (–)	81 (–)	79 (2.75)	81 (0.00)
$N = 85$	Time (s)	0.30	0.62	0.42	0.51	0.51	563.43	26.33	163.32

Table 2. Multi-view triangulation. Only our algorithm amongst all methods returns error bounds

Thanks for your attention

Homepage: https://sites.google.com/view/dzungdoan/home

Paper link: https://arxiv.org/abs/2201.10110

Source code: https://github.com/dadung/HQC-robust-fitting

Session: Poster 1.1, ID: 42a